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$$=\frac{ab}{\sqrt{(1-m^2\sin^2\theta)}}(m^2\sin^2\varphi\sin\theta\cos^2\theta+\sin\theta\cos^2\varphi-m^2\sin^3\theta\cos^2\varphi)=\frac{Aab\sin\theta}{\Delta(\theta)}.$$

$$\frac{c^2}{z}(x^2/a^4+y^2/b^4+z^2/c^4)=\frac{c}{\sin\theta\,\Delta(\varphi)}\Big(\frac{\sin^2\varphi(1-m^2\sin^2\theta)}{a^2}+\frac{\cos^2\theta\cos^2\varphi}{b^2}$$

$$+\frac{\sin^2\theta(1-n^2\sin^2\varphi)}{c^2}\Big)=\frac{1}{a^2b^2c\sin\theta\,\Delta(\varphi)}[b^2c^2\sin^2\varphi(n^2+m^2\cos^2\theta)$$

$$+a^2c^2\cos^2\theta\cos^2\varphi+a^2b^2\sin^2\theta(m^2+n^2\cos^2\varphi)]=\frac{B}{a^2b^2c\sin\theta\,\Delta(\varphi)}.$$

$$\therefore\int\frac{ds}{p}=\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{abc\,\Delta(\theta)\,\Delta(\varphi)}=\frac{\pi}{6abc}(a^2b^2+a^2c^2+b^2c^2).$$

$$\therefore\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{\Delta(\theta)\,\Delta(\varphi)}=\tfrac{1}{6}\pi(a^2b^2+a^2c^2+b^2c^2).$$

Also $\displaystyle\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{\Delta(\theta)\,\Delta(\varphi)}=\tfrac{1}{3}(a^2b^2+a^2c^2+b^2c^2)[E(m)F(n)$

$+F(m)E(n)-F(m)F(n)]$.

$\therefore$ $E(m)F(n)+F(m)E(n)-F(m)F(n)=\frac{1}{2}\pi$, Legendre's Theorem.

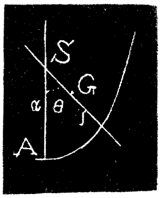
MECHANICS.

155. Proposed by **M. E. GRABER**, Graduate Student, Heidelberg University, Tiffin, Ohio.

A parabolic curve is placed in a vertical plane with its axis vertical aud vertex downwards, and inside of it, and against a peg in the focus, and against the concave arc, a smooth uniform and heavy beam rests. Find the position of equilibrium.

Solution by **G. W. GREENWOOD**, A. B., Professor of Mathematics, McKendree College, Lebanon, Ill., and **E. L. SHERWOOD**, Shady Side Academy, Pittsburg, Pa.

Let the equation to the parabola be $2a/r=1+\cos\theta$, and let the length of the rod be $2l$. We must have $2l>a$. When the rod makes an angle θ with the axis, the depth of its center of gravity below the focus is



$$n=(r-l)\cos\theta=\Big(\frac{2a}{1+\cos\theta}-l\Big)\cos\theta.$$

In a position of equilibrium, n is a maximum or a minimum.

$$\therefore\ \sin\theta\Big(\frac{2a}{(1+\cos\theta)^2}-l\Big)=0\quad\therefore\ \theta=0,\text{ or }\cos\theta=\sqrt{(2a/l)}-1.$$

We must take the positive sign with the radical.

Since $2l>a$, $\sqrt{(2a/l)}<2$, and the values of θ obtained from the last equation are real, but in order that θ be not greater than 90°, an obvious requirement in this problem, we must have $l<2a$. In any case, the position given by $\theta=0$ is unstable. The two values of θ given by the second equation give the positions of stable equilibrium when $2a>l>\frac{1}{2}a$.

Also solved by *G. B. M. ZERR.*

156. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, Eng.

Three perfectly elastic particles start from the cusp of a smooth cycloid (axis vertical, vertex down) at intervals of t seconds. How long will it be to the nth collision ?

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Since the curve is a smooth cycloid and the particles perfectly elastic and of equal mass, whenever collision takes place the two particles that collide have the same velocity. The reaction of one upon the other from impact is the same as though one particle passed through the other without either being affected.

Let mt $(m>2)$ be the time it takes a particle to go from cusp to cusp, and a, the radius of the generating circle. Then $m=(2\pi/t)\sqrt{(a/g)}$. After the first particle has arrived at the opposite cusp, the second is t and the third $2t$ seconds behind it.

$\therefore$ Time to first collision is $mt+t/2=(2m+1)t/2$ seconds.
Time to second collision is $mt+t=(2m+2)t/2$ seconds.
Time to third collision is $mt+3t/2=(2m+3)t/2$ seconds.
Time to fourth collision is $mt+t+mt-t+t/2=(4m+1)t/2$ seconds.
Time of fifth collision is $(4m+2)t/2$ seconds.
Time of sixth collision is $(4m+3)t/2$ seconds; etc.

$\therefore$ The collisions take place in sets of three. The pth set is $(2pm+1)t/2$, $(2pm+2)t/2$, $(2pm+3)t/2$. If the nth collision is the 1st, 4th, 7th, ...$(3p-2)$th, then $n=3p-2$ or $p=(n+2)/3$, and it takes place in $[2(n+2)m+3]t/6$ seconds; if it is the 2nd, 5th, 8th, ...$(3p-1)$th, it takes place in $[2(n+1)m+6]t/6$ seconds; if it is the 3rd, 6th, 9th, ...$3p$th, it takes place in $(2nm+9)t/6$ seconds.

Also solved by *G. W. GREENWOOD.*

157. Proposed by T. W. WRIGHT, Professor of Mathematics, Schenectady, N. Y.

Explain why a waterfall h feet high can support a column $2h$ feet high.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let w=weight of a cubic foot of water. A column of water 1 foot square falling through h feet weighs hw pounds; to this must be added the kinetic energy of a cubic foot of water falling through h feet; this equals $wv^2/2g$, but $v^2=2gh$.

$\therefore$ kinetic energy$=wh$. $\therefore$ $wh+wh=2wh=w(2h)$.